From (1), z=.00066. s=6416.9M. The ultimate strength of hard steel is 240000 lb./in.^{2}.

$\therefore s=240000/f=240000/6=40000$ lb./in.^{2}

$\therefore 40000=6416.9M$. $\therefore M=6.2335$.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

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159. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

Show that if the equation $y^3=2x^2-1$ be possible in integers, $y=24n^2-1$, or $2n^2-1$, and find three solutions.

Solution by the PROPOSER.

The equation may be written $(y+1)(y^2-y+1)=2x^2$.

Since y^2-y+1 is always odd, it is evident that $y+1$ must be even. Since $y^2-y+1=(y+1)(y-2)+3$ it is evident that $y+1$ and y^2-y+1 can have no common factor but 3. Therefore we have the following possibilities for y: $y=2\times3m^2-1$, or $y=2n^2-1$.

Since y^3 is represented by the form $2x^2-1$, 2 must be a quadratic residue of y. Therefore $y=8a\pm1$, and this is possible in the first expression only when $m=2n$. Then either $y=24n^2-1$ or $2n^2-1$.

Substituting these values of y in the original equation, we have

$$192n^4-24n^2+1=r^2 \text{ or } 4n^4-6n^2+3=r^2.$$

The first equation has the solution $n=0$ and 1 which give $y=-1$, $x=0$; $y=23$, $x=78$.

The second equation has the solution $n=1$, which gives $y=1$, $x=1$.

AVERAGE AND PROBABILITY.

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198. Proposed by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill.

Find the average length of a hole at random through a given cylinder.

No solution of this problem has been received.

199. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

A circle is inscribed in a given square. Two points are taken at random within the square but without the circle. What is the chance the distance between the points does not exceed the side of the square?

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

This is the same as 196, but as there is a distance less than the side of the square when both points are taken one each in opposite corners, it is

desirable that another solution be presented. Professor Carmichael called my attention to this error.

As a solution by Calculus would be very long and tedious and well nigh impossible, we will use a formula given in the *Encyclopedia Britannica.* It is as follows:

If p is the probability of a certain condition being fulfilled by n points within an area A, p' the probability when they fall on area $A+B$ (B without A), p_1 the probability when one point falls on B and the rest on A, then $(p'-p)A=nB(p_1-p)$.

In the problem, $A=\pi a^2$, $B=(4-\pi)a^2$, $n=2$, where $2a$=side of the square. Now $p'=\pi-\frac{13}{6}$ (MONTHLY, Vol. III, No. 11, page 285); $p=1$ for both points on A.

$\therefore (\pi-\frac{13}{6}-1)\pi a^2=2(4-\pi)(p_1-1)a^2$.

$\therefore p_1=\dfrac{6\pi^2-31\pi+48}{12(4-\pi)}$, the probability that the distance is less than the side of the square when one point is within, the other without the circle.

Similarly, $(p'-P)B=2A(p_1-P)$.

$\therefore P=\dfrac{2\pi p_1-p'(4-\pi)}{3\pi-4}$. $P=\dfrac{15\pi^2-76\pi+104}{3(4-\pi)(3\pi-4)}$.

MISCELLANEOUS.

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176. Proposed by WM. E. HEAL, Coffeyville, Kansas.

In Grassman's *Extensive Algebra*, $e_1e_2=-e_2e_1$. If $e_1=e_2$, $e_1^2=-e_1^2=0$. In quaternions, $ij=-ji$, $i^2j=i.ij=ik=-j$, $i^2=-1$. Reconcile these apparently divergent results.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

i, j, k are unit vectors at right angles to each other. By definition: The effect of any unit vector acting as a multiplier upon another at right angles to it, is the turning of the latter in a positive direction (counter clock wise) in a plane perpendicular to the operator or multiplier through an angle $\frac{1}{2}\pi$. Hence the product or quotient of two unit vectors at right angles is a unit vector perpendicular to their plane. In a positive direction i operating on j produces k, or $ij=k$. Similarly, $jk=i$, $ki=j$. j operating on i in a negative direction produces $-k$. Therefore, $ji=-k$. Similarly, $ik=-j$, $kj=-i$.

The operation of multiplying here is not a numerical product, and hence it is a geometric multiplication, and not an algebraic process.

As the effect of i, j, k as operators is to turn a line from one direction into another which differs from it by 90°, they are called quadrantal versors.

Now $ij=k$, $ik=-j$. $\therefore i.ij=-j=-1.j=i^2j$.

$\therefore i.i=i^2=-1$. Also, $ij=k$, $ji=-k$. $\therefore ij=-ji$.

(See Hardy's *Elements of Quaternions*, pp. 40-48.)